→ Extreme Value Thm. → on closed interval if $f(x)$ is continuous then has a min + max value

→ local extrema at $f'(x) = 0$

→ Critical Point when $f'(x) = 0$ or is undefined may or may not be extre

→ Mean Value Thm for Derivatives → continuous on $[a, b]$ differentiable on (a, b) then at least one pt.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

→ increasing / decreasing function $f'(x) > 0$ or $f'(x) < 0$

→ Antiderivative

→ graphs of $f'(x)$, $f''(x)$, ω $f(x)$

→ concavity: up if $f''(x) > 0$ down $f''(x) < 0$

→ point of inflection: change in concavity

→ First + Second derivative tests

↓ $f'(x)$ changes sign

↘ $f'(c) = 0$ + $f''(c) < 0 \Rightarrow$ local max

$f'(c) = 0$ + $f''(c) > 0 \Rightarrow$ local min

→ Optimization

→ Linearization $f(a) + f'(a)(x-a) = L(x)$

→ differentials $dy = f'(x)dx$

→ Related Rates – method p. 246

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\dfrac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

$A = \pi r^2$

$\dfrac{\partial A}{\partial t} = \underline{2\pi r} \dfrac{dr}{dt}$

$\dfrac{\partial A}{\partial t} = C\ (-0.1)$

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

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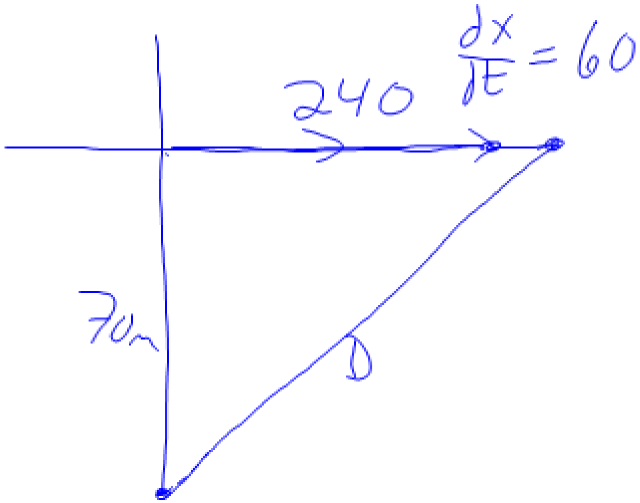

Find $\frac{dD}{dt}$

$$70^2 + x^2 = D^2$$

$$2x\frac{dx}{dt} = 2D\frac{dD}{dt}$$

$$2(240)(60) = 2(250)\frac{dD}{dt}$$

$$\boxed{\frac{dD}{dt} = 57.6 = A}$$

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

$$A = \frac{1}{2} b \cdot h$$

$$\frac{dA}{dt} = \frac{1}{2}\left[b\frac{dh}{dt} + h\frac{db}{dt}\right]$$

$$= \frac{1}{2}\left[-3b + 3h\right]$$

$$= \frac{1}{2}\left[3h - 3b\right]$$

$$= \frac{3}{2}\left[h - b\right]$$

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.